Rusczyk, R. (2007), Introduction to Geometry (2nd Edition), AoPS Incorporated, Alpine, CA.

3.6 Isosceles and Equilateral Triangles

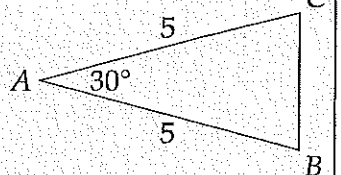
Problems

Problem 3.15: In the diagram, $AB = AC = 5$ and $\angle CAB = 30°$.

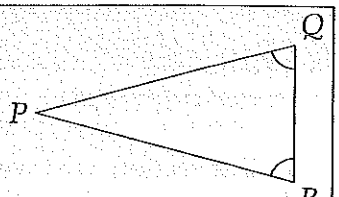
(a) Let M be the midpoint of $\overline{BC}$. Draw $\overline{AM}$. Prove that $\triangle ACM \cong \triangle ABM$.

(b) Find $\angle AMB$.

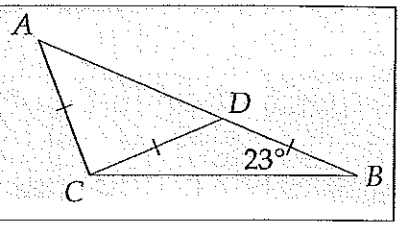
(c) Find $\angle ACB$.



Problem 3.16: Prove that if $\angle PQR = \angle PRQ$, then $PR = PQ$.



Problem 3.17: In the diagram, $AC = CD = DB$, and $\angle B = 23°$. Find $\angle A$.



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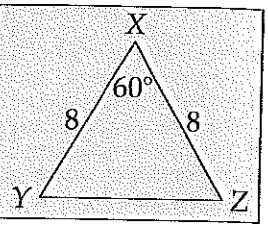
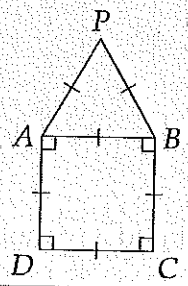
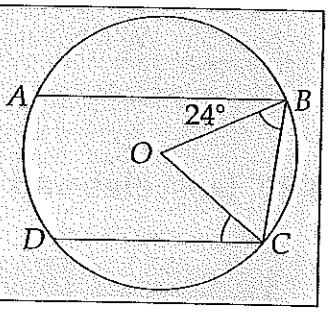
Problem 3.18: In the diagram, $XY = XZ = 8$ and $\angle X = 60°$. Find YZ.



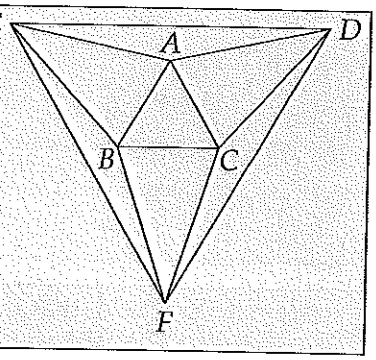
Problem 3.19: Find $\angle PBD$.



Problem 3.20: In the diagram, O is the center of the circle, $\overline{AB} \parallel \overline{CD}$, $\angle ABO = 24°$, and $\angle OBC = \angle OCD$. Find $\angle BOC$.



Problem 3.21: In the diagram at right, $AB = BC = AC$ and $AE = EB = BF = FC = CD = DA$ such that $\triangle ABC$ is completely inside $\triangle DEF$. Prove that $DE = EF = DF$.



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EXERCISES

3.6.1 In $\triangle PQR$, $PQ = PR$ and $\angle P = 43°$. Find $\angle Q$.

3.6.2 Prove that if $AB = AC$ in $\triangle ABC$, then $\angle ABC = \angle ACB$. (Note: You cannot simply state that the triangle is isosceles, so the base angles are equal. You are asked here to prove this fact.)

3.6.3

(a) Prove that if the three sides of a triangle are equal in length, then all three angles of the triangle have measure 60°.

(b) Prove that if the three angles of a triangle are equal, then all three sides of the triangle have the same length.

Note that for this problem, you are not allowed to state that the triangle is equilateral as your proof. You must prove the facts about equilateral triangles that you learned in the text.

3.6.4 Two angles of an equilateral triangle have measures $3x + 27°$ and $2y - 4°$. Find $x + y$.

3.6.5 Point O is the center of the circle in the diagram on the left below. Find $\angle AOB$ if $\angle OAB = 70°$. **Hints:** 79

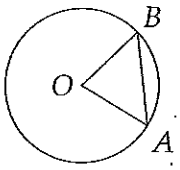

Figure 3.3: Diagram for Problem 3.6.5

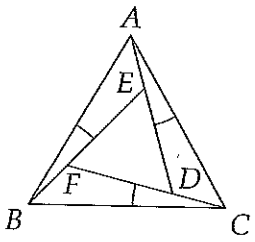

Figure 3.4: Diagram for Problem 3.6.6

3.6.6 Triangle ABC is equilateral in the diagram on the right above. Points D, E, and F are in triangle ABC such that $\angle CAD = \angle ABE = \angle BCF$. D lies on $\overline{CF}$, E lies on $\overline{AD}$, and F lies on $\overline{BE}$. Prove that triangle DEF is equilateral. **Hints:** 192, 574

3.6.7 In the diagram at right, $VW = VX$ and $\overline{WX} \parallel \overline{YZ}$.

(a) Prove that $WY = XZ$. **Hints:** 178, 518

(b) Prove that $YX = WZ$. **Hints:** 315