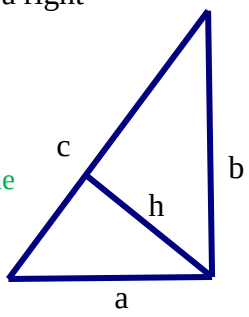
Name: Problem Set 3

Write your answers with good explanations in a separate sheet of paper.

1. Theo tied his dog, Flash, to a pole in the middle of the yard using a 10 ft. leash. Flash dug holes in the yard everywhere he could reach. Theo had to reseed the part of the lawn that Flash destroyed. Grass seed costs $1.40 per package and covers 50 square feet. What did it cost Theo to reseed the lawn?

The shape of the lawn that Flash destroyed would be a circle with radii equal to 10ft because the furthest distance that Flash could reach is 10ft, which was the length of the leash. Hence, the area of lawn that Flash destroyed was, $\pi r^2 = 100\pi \approx 314$ square feet. Since one pack of grass seed can cover 50 square feet, the amount of pack Theo needed to reseed the lawn is $\frac{Area}{50\ square\ feet/pack}$ =314/50=6.28 ≈7. It was rounded up to 7 because people didn't sell .28 packs. Thus, it cost Theo $9.80 (7*$1.40) to reseed the lawn since each pack costs $1.40.

2. What is the length of the altitude from the right angle to the hypotenuse in a right triangle with legs *a* and *b* and hypotenuse *c*?

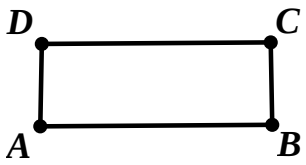


Based on the area of a triangle formula, $A = \frac{1}{2}bh$ and two legs of a right triangle perpendicular to each other, so the area of a right triangle with legs *a* and *b* is $\frac{1}{2}ab$ if I choose *a* as the base. If I choose the hypotenuse, *c*, as the base and denote the length of the altitude from the right angle to the hypotenuse as *h*, then A= $\frac{1}{2}ch$. The area of this triangle remains the same no matter which side I choose as the base since the space doesn't change. Thus, $\frac{1}{2}ab = \frac{1}{2}ch$. So, h = $\frac{ab}{c}$.

3. How many sides does a regular polygon have if all its interior angles measure 162 degrees?

Let's say that the number of sides this regular polygon has is n. The sum of interior angles formula of a polygon says that the sum is equal to (n-2)180, n is the number of sides. So, the sum of interior angles of this regular polygon is (n-2)180. We all also know that this regular polygon has all its interior angels measure 160 degrees, so the sum of all the interior angles is #of sides * angle measure of one interior angle = n* 162. Therefore, n*162 = (n-2)180, n=20. Hence, a regular polygon that has all its interior angles measure 162 degrees has 20 sides.

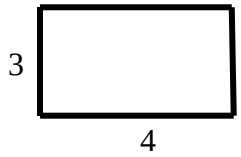
4. Let *R* be a rectangle. How many circles in the plane of *R* have a diameter both of whose endpoints are vertices of *R?*



A rectangle has four vertices. The number of ways to form a diameter with both endpoints are vertices of R is 6: AB, AC, AD, BC, BD, CD. Hence, we can form 6 circles in the plane of R have a diameter both of whose endpoints are vertices of R.

5. A 3×4 paper rectangle can be curled to form a cylinder in two ways. Find the ratio of the volumes of the two possible cylinders.

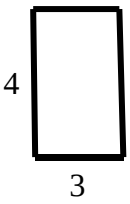
Case 1:



We can curl the 3* 4 paper in a way that 4 is the circumference of the base and 3 be the measure of the height. Since 4 is the circumference and the base of a cylinder is a circle, $2\pi r = 4$, $r = 2/\pi$. Based on the volume formula for a cylinder $V = \pi r^2 h$, $V_1 = 12/\pi$.

Case 2:

We can curl the 3* 4 paper in a way that 3 is the circumference of the base and 4 be the measure of the height. Since 3 is the circumference and the base of a cylinder is a circle, $2\pi r = 3$, $r = 3/(2\pi)$. Based on the volume formula for a cylinder $V = \pi r^2 h$, $V_2 = 9/\pi$.



Thus, the ratio of the volumes of the two possible cylinders is 12:9 or 4:3.

6. The right triangle with sides of length 5, 12, and 13 has the property that its area is equal to its perimeter. Find another right triangle, also with sides of integral length that has this property.

6, 8, 10 is another pair of right triangle. The area is 6*8/2=24 and its perimeter=6+8+10=24.

In order to find more pairs of right triangles with this property, you can set up equation of area and perimeter. Let's denote the sides of the right triangle is a, b, and c (hypotenuse). Then $\frac{1}{2}*a*b=a+b+c$, and $c=\sqrt{a^2+b^2}$. At the end, you will find out a relationship between a and b.

7. Given 30-60-90 °△ABC with inscribed squares of side lengths x and y, find x/y in simplest radical form.